

Comment on “Evidence for nontrivial ground-state structure of $3d \pm J$ spin glasses”

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In a recent *Letter* [1], Hartmann presented results for the structure of the degenerate ground states of the three-dimensional $\pm J$ spin glass model obtained using a genetic algorithm. Further work was carried out using similar techniques [2]. In this *Comment*, I argue that the method does not produce the correct thermodynamic distribution of ground states and therefore gives erroneous results for the overlap distribution. I present results of simulated annealing [3] calculations using different annealing rates for cubic lattices with $N = 4^3$ spins. The disorder-averaged overlap distribution exhibits a significant dependence on the annealing rate, even when the energy of the lowest state has converged. For fast annealings, moments of the distribution are similar to those presented in Ref. [1]. However, as the annealing rate is lowered, they approach the results obtained by Berg, Hansmann, and Celik [4] using a multi-canonical Monte Carlo method. This shows explicitly that care must be taken not only to reach states with the lowest energy but also to ensure that they obey the correct thermodynamic distribution, i.e., that the probability is the same for reaching any of the ground states.

The simulated annealing procedure was carried out starting at inverse temperature $\beta = J/T = 0$ and increasing β in steps of $\Delta\beta = 0.01$ up to $\beta = 3$. For each β , one or several Monte Carlo updating cycles were carried out. Each cycle consisted of N single-spin Metropolis updates with the spins chosen at random. The number of updating cycles per β value was chosen as 2^n , with n hence defining the logarithm of the inverse annealing rate. The state with the lowest energy last reached in the process was stored. For sufficiently slow annealings (sufficiently large n), this procedure is guaranteed to give a ground state, with the same probability for reaching all the ground states. On the order of $3 - 10 \times 10^4$ random configurations with equal amounts of $+J$ and $-J$ interactions were studied for $n = 0, 1, \dots, 5$, and for each configuration 100 replicas were considered. Out of these, only those with the lowest energy were used in calculating averages. On average, 80% of the replicas reached the lowest energy for $n = 0$, and for $n = 4$ this fraction exceeded 99%.

Figure 1 shows the dependence of some disorder-averaged quantities on n . For the small system size used, the correct ground state energy is reproduced already for $n = 0$ (i.e., the fastest annealing considered), as can be seen in the comparison with the result obtained by Pál [5] (which also agrees with the less accurate result of Ref. [1]). However, the moments of the overlap distribution still exhibit a strong dependence on n . For $n = 0$, both the average $\langle |q| \rangle$ and the variance $\sigma^2(|q|)$ of the absolute value of the overlap q are close to the results presented in Ref. [1]. As n is increased, the values change in a way reflecting a reduction in the overlap distribution weight for small q . They approach the results of multi-canonical Monte Carlo simulations [4]. This behavior indicates that faster annealings reach states with the lowest energy but not with the same probability for all states. Clearly, this is due to the fast process not exploring the full configuration space, and therefore not sufficiently often reaching regions where the density of ground states is high. The genetic algorithm used in Refs. [1,2] can also be expected to be affected by such behavior, as there is nothing in the process that guarantees an equal probability for reaching all ground states. Hence, the results obtained with this method do not reflect the thermodynamic behavior of the model and the conclusions reached in Ref. [1] for the ground state structure must be questioned.

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- [1] A. K. Hartmann, *Europhys. Lett.* **40**, 429 (1997).
 - [2] A. K. Hartmann, cond-mat/9804325 (to appear in *Europhys. Lett.*); cond-mat/9806114 (to appear in *Phys. Rev. E*); cond-mat/9808197; cond-mat/9810037.
 - [3] S. Kirkpatrick, S. Gelatt, and M. P. Vecchi, *Science* **220**, 671 (1983).
 - [4] B. A. Berg, U. E. Hansmann, and T. Celik, *Phys. Rev. B* **50**, 16444 (1994).
 - [5] K. F. Pál, *Physica A* **223**, 283 (1996).

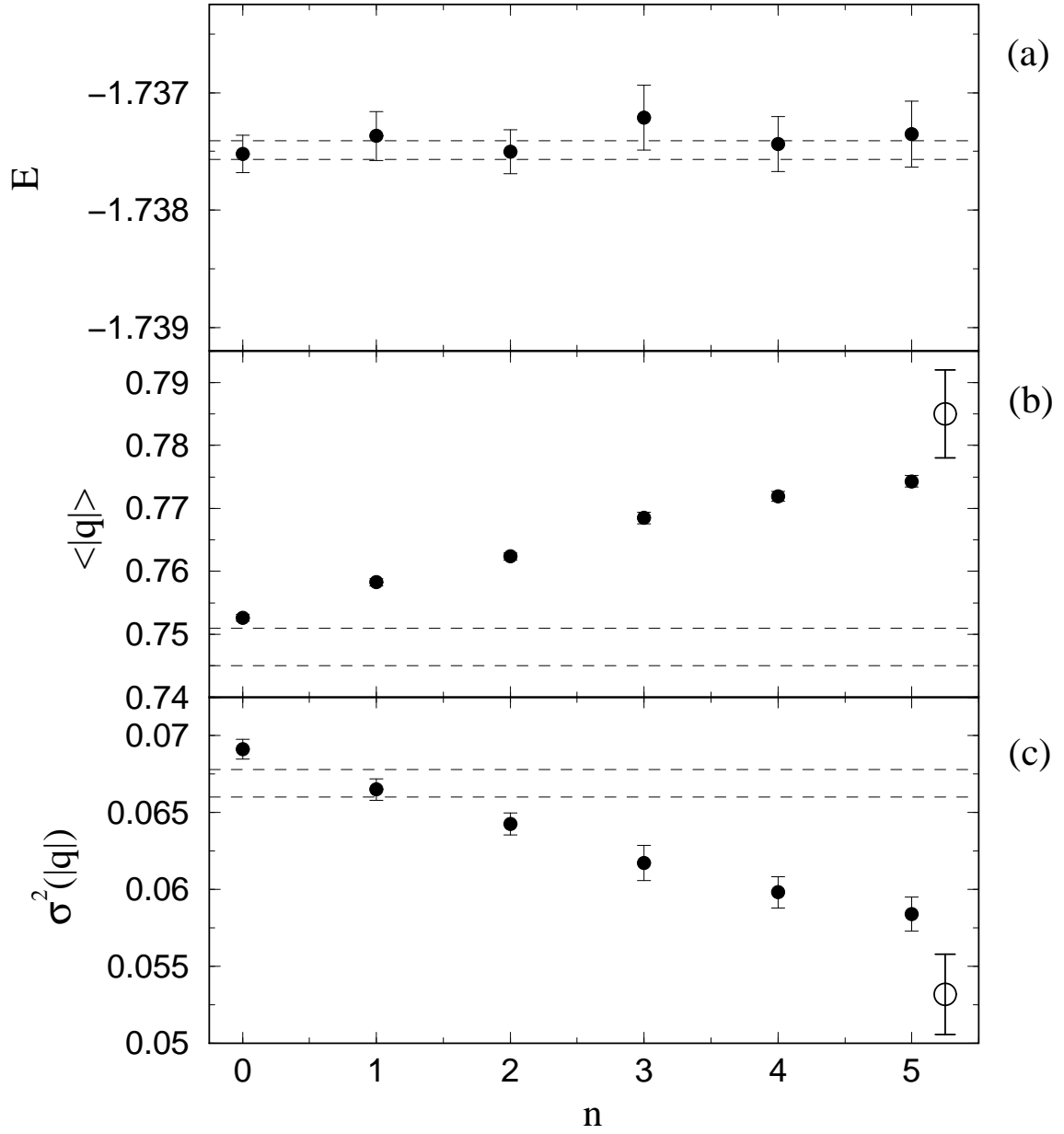


FIG. 1. Disorder-averaged quantities calculated using lowest-energy states obtained in simulated annealings with 2^n updating cycles per temperature. (a) Energy. The result (average \pm one error bar) of Ref. [5] is indicated by the dashed lines. (b) Average and (c) variance of the overlap distribution. The results of Ref. [1] are indicated by the dashed lines. The multi-canonical Monte Carlo results [4] are shown as the open circles.